Course Syllabus

General Information

Course name	Functional analysis
Programme	Mathematics
Level of studies (BA, BSc, MA, MSc, long-cycle MA)	BA
Form of studies (full-time, part-time)	full-time
Discipline	Mathematics
Language of instruction	English

Course coordinator/person responsible	dr hab. A. Zapała

Type of class *(use only the types mentioned below)*	Number of teaching hours	Semester	ECTS Points
lecture	30	6	5
tutorial			
classes	30	6	
laboratory classes			
workshops			
seminar			
introductory seminar			
foreign language classes			
practical placement			
field work			
diploma laboratory			
translation classes			
study visit			

Course pre-requisites	Mathematical analysis (differential and integral calculus), Linear algebra (the notion of a vector space), Topology of metric spaces (definition of a distance, separable and complete metric spaces), The Lebesgue measure and integral

I. Course Objectives

Learning of fundamental notions and classical examples of Banach spaces
Study of the most important theorems on linear functionals and linear operators in Banach spaces
Presentation of selected applications of functional analysis in other branches of mathematics

II. Course learning outcomes with reference to programme learning outcomes

Symbol	Description of course learning outcome	Description of course learning outcome
KNOWLEDGE		
W_01	Students understand definitions of norms, normed linear spaces and metrics generated by norms	K_W01, K_W03
W_02	Students get acquainted with the most important examples of Banach spaces	K_W04
W_03	Students get acquainted with the classical theorems on linear functionals and linear operators in Banach spaces	K_W02, K_W03, K_W04, K_W05
W_04	Students get acquainted with the selected applications of the theory of Banach and Hilbert spaces	K_W03, K_W04, K_W05
SKILLS		
U_01	Students ascertain whether given functions define norms in vector spaces	K_U02, K_U03, K_U04, K_U09
U_02	Students investigate completeness and separability of normed linear spaces	K_U03, K_U04, K_U10
U_03	Students apply theorems of functional analysis to solve problems in other branches of mathematics	K_U01, K_U02, K_U05, K_U06
SOCIAL COMPETENCIES		
K_01	Students precisely formulate questions to deepen the understanding of the subject and complement the missing elements of reasoning	K_K01, K_K05

III. Course Content

Linear spaces and metric spaces.
Separability and completeness of a metric space.
Normed linear spaces: properties of the norm, the metric generated by the norm, examples of normed linear spaces, finite dimensional normed linear space, equivalent norms.
Banach spaces: definition, examples of classical sequential and functional Banach spaces.
Linear operators and functionals in normed linear spaces: continuity and boundedness of linear operators, the norm of a linear operator. The space of continuous linear operators.
The most important theorems on linear operators and linear functionals in Banach spaces: the Hahn-Banach theorem, sequences of continuous linear operators - the Banach-Steinhaus theorem, the open mapping Banach theorem and closed graph Banach theorem. Invertible operators.
The notion of the conjugate space, examples of dual spaces. Reflexive spaces.
Scalar product, the Schwarz inequality and the Minkowski inequality.
The norm and metric generated by the scalar product.
Definition and examples of unitary spaces.
Orthogonal systems. The Schmidt orthonormalization method.
Fourier coefficients and series expansions.
The Bessel inequality and the Riesz-Fisher theorem.

Complete and closed orthogonal systems, the Parseval identity. Theorem on a projection onto the closed convex set in a Hilbert space. Orthogonality, orthogonal projection onto a closed subspace of a Hilbert space. Linear operators and functionals in a Hilbert space. Informations concerning linear functionals and the dual space to a Hilbert space.

IV. Didactic methods used and forms of assessment of learning outcomes

Symbol	Didactic methods *(choose from the list)*	Forms of assessment *(choose from the list)*	Documentation type *(choose from the list)*
KNOWLEDGE			
W_01	Conventional lecture	Test or Exam	Evaluated test or Protocol
W_02	Conventional lecture	Test or Exam	Evaluated test or Protocol
W_03	Conventional lecture	Test or Exam	Evaluated test or Protocol
W_04	Conventional lecture	Test or Exam	Evaluated test or Protocol
SKILLS			
U_01	Practical classes	Written test	Evaluated test
U_02	Practical classes	Written test	Evaluated test
U_03	Practical classes	Written test	Evaluated test
SOCIAL COMPETENCIES			
K_01	Problem-Based Learning	Test	Evaluated test

V. Grading criteria, weighting factors.

LECTURE: The written exam consists of two parts: 1) practical (60%) - verifying the ability to apply the knowledge in practice, 2) theoretical (40%) - checking theoretical knowledge. Detailed criteria are given to students with each edition of the subject.

CLASSES: At least 80% of attendance is required. Two tests together constitute the final grade:
86 – 100% excellent, 71 – 85% very good, 56 – 70% good, 41 – 55% satisfactory, 31 – 40% sufficient, less than 30% fail

VI. Student workload

Form of activity	Number of hours
Number of contact hours (with the teacher)	60+30(consultations)
Number of hours of individual student work	60

VII. Literatura

Basic literature
J. Musielak, Wstęp do analizy funkcjonalnej, PWN, Warszawa 1989. J. Conway, A Course in Functional Analysis, Springer-Verlag, New York 1997. A. Alexiewicz, Analiza funkcjonalna, PWN, Warszawa 1969. L. A. Lusternik, W. I. Sobolew, Elementy analizy funkcjonalnej, PWN, Warszawa 1959. W. Rzymowski, Macierze i operatory, Wyd. UMCS, Lublin 2005.
Additional literature
S. Prus, A. Stachura, Analiza funkcjonalna w zadaniach, PWN, Warszawa 2007.